

# Lunar Laser Ranging - A Science Tool for Geodesy and General Relativity

*Jürgen Müller*

Institut für Erdmessung, Leibniz Universität Hannover, Germany



Leibniz  
Universität  
Hannover



# Acknowledgement

Work has been supported by



**DFG Research Unit FOR584**  
**Earth Rotation and Global Dynamic Processes**  
**(computations by Liliane Biskupek)**



**and the Centre of Excellence QUEST**  
**(Quantum Engineering and Space-Time Research)**

# Contents

---

## Introduction

- Motivation

## Lunar Laser Ranging

- Data (distribution and accuracy)
- Analysis

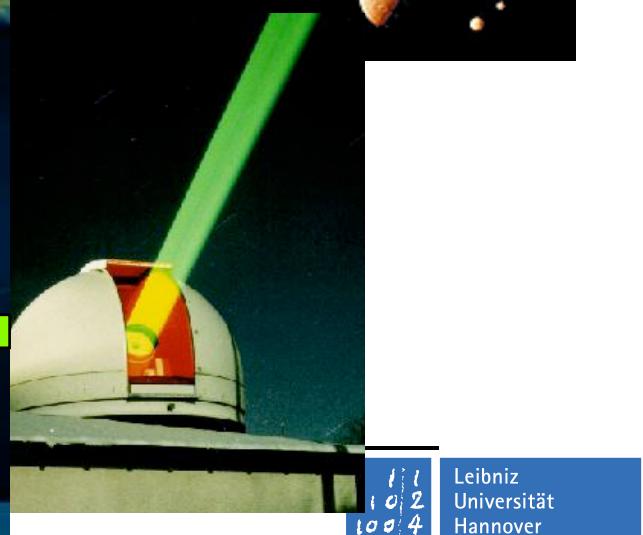
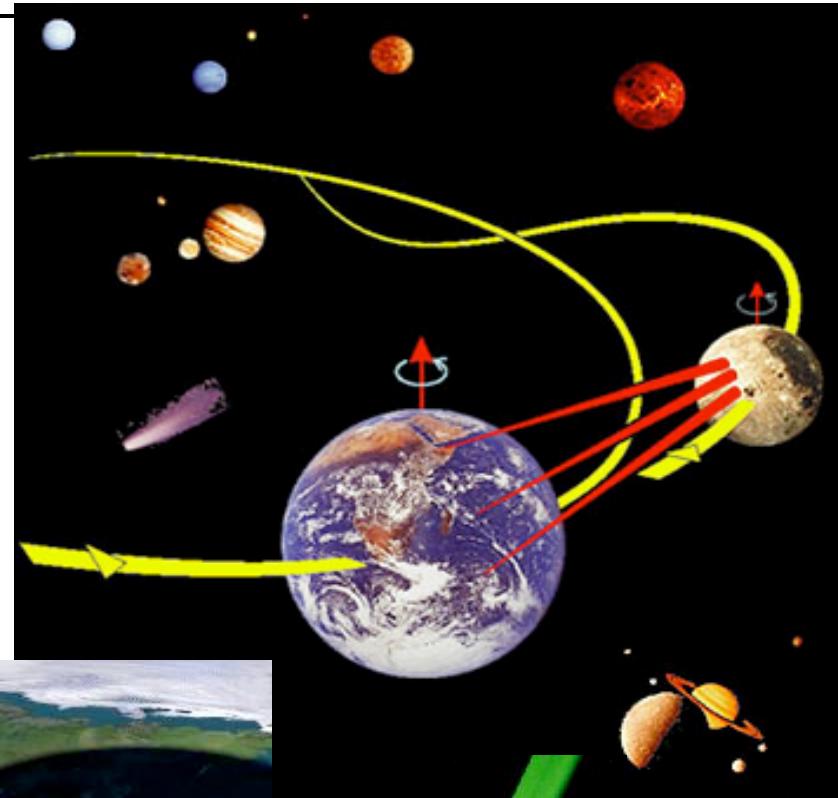
## Relativity Tests

## Conclusions

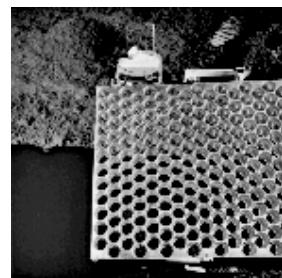
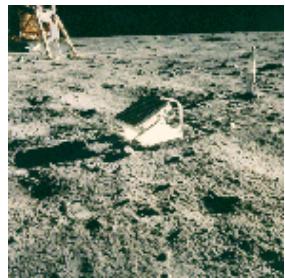
- Future capabilities

# Lunar Laser Ranging (LLR)

- 38 years of observations
- Modelling so far at cm-level
- Long-term stability (e.g., orbit)
  - Earth-Moon dynamics
  - Relativity parameters



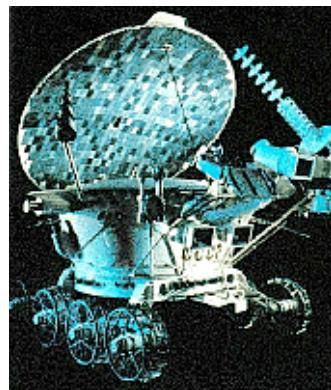
# Retro-Reflectors on the Moon



Apollo 11  
July 1969

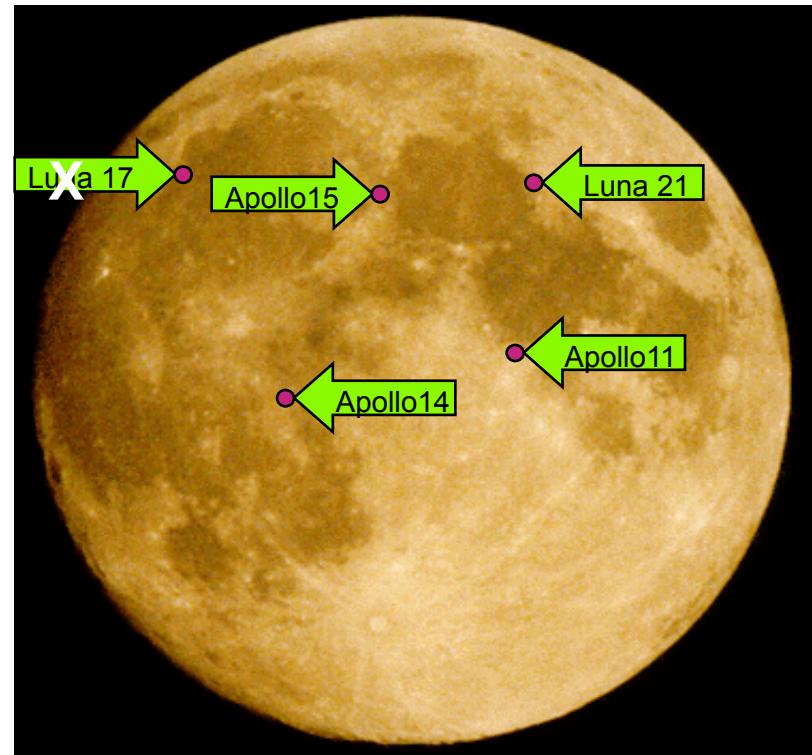
Apollo 14  
Jan./Feb. '71

Apollo 15  
Jul./Aug. '71



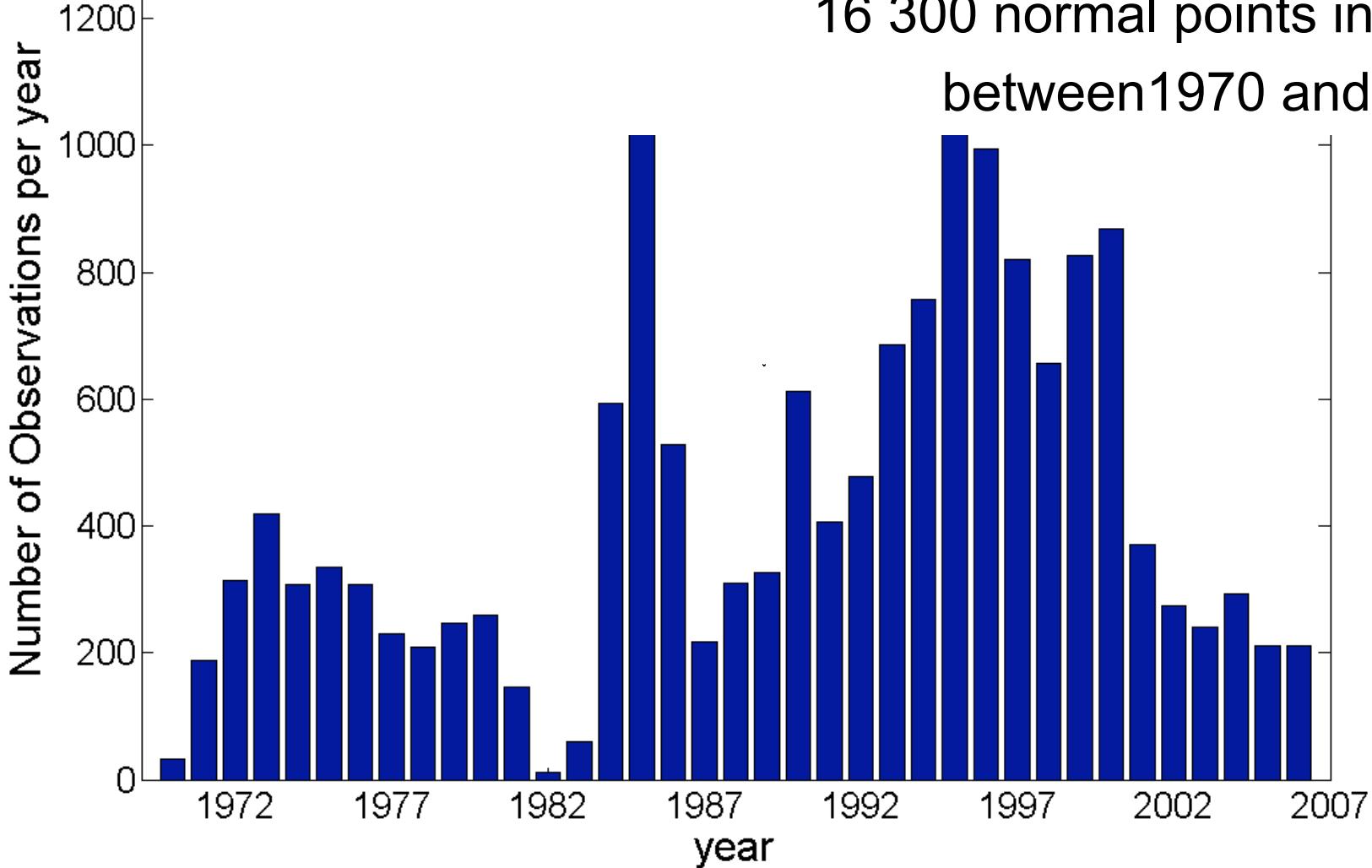
Luna 17  
Nov. '70...

Luna 21  
Jan. '73...

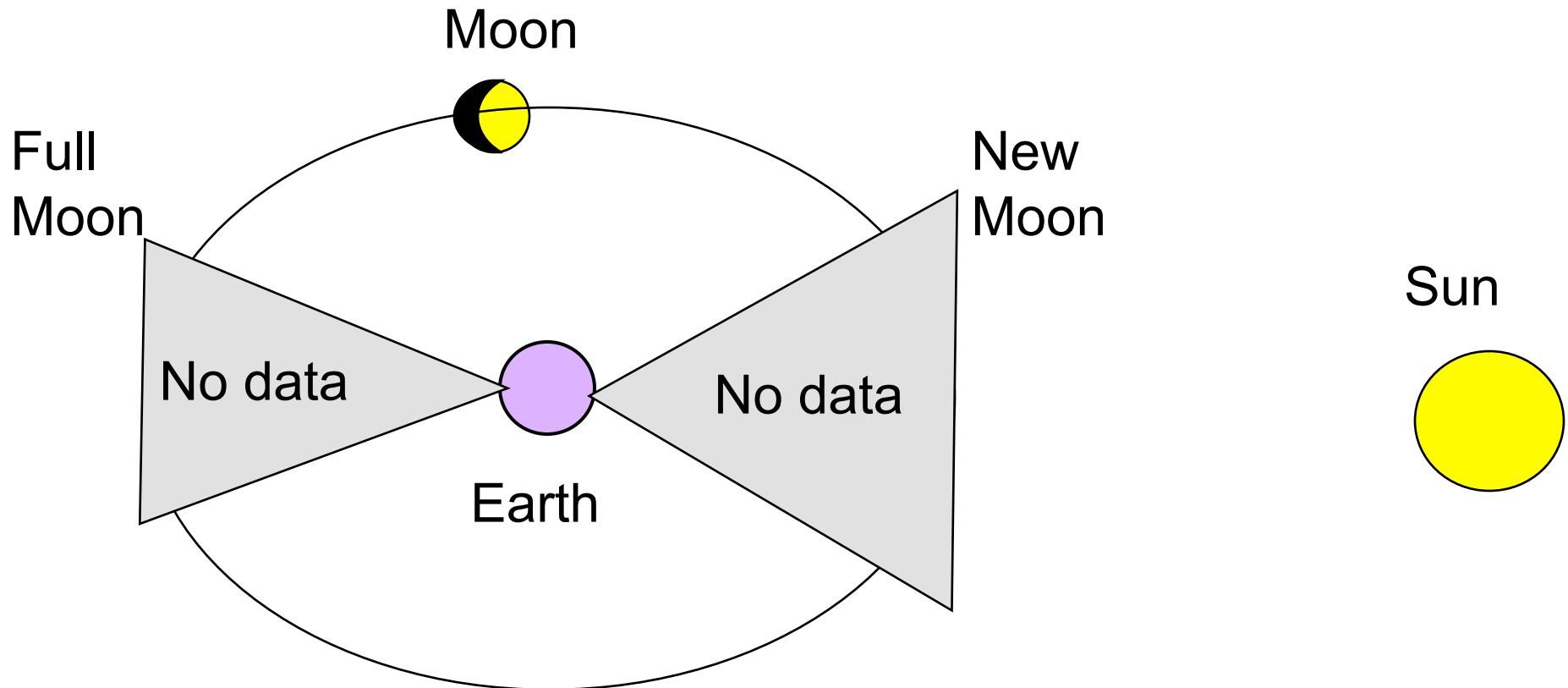


# LLR Observations per Year

Number of observations; annually averaged;  
16 300 normal points in total,  
between 1970 and 2008

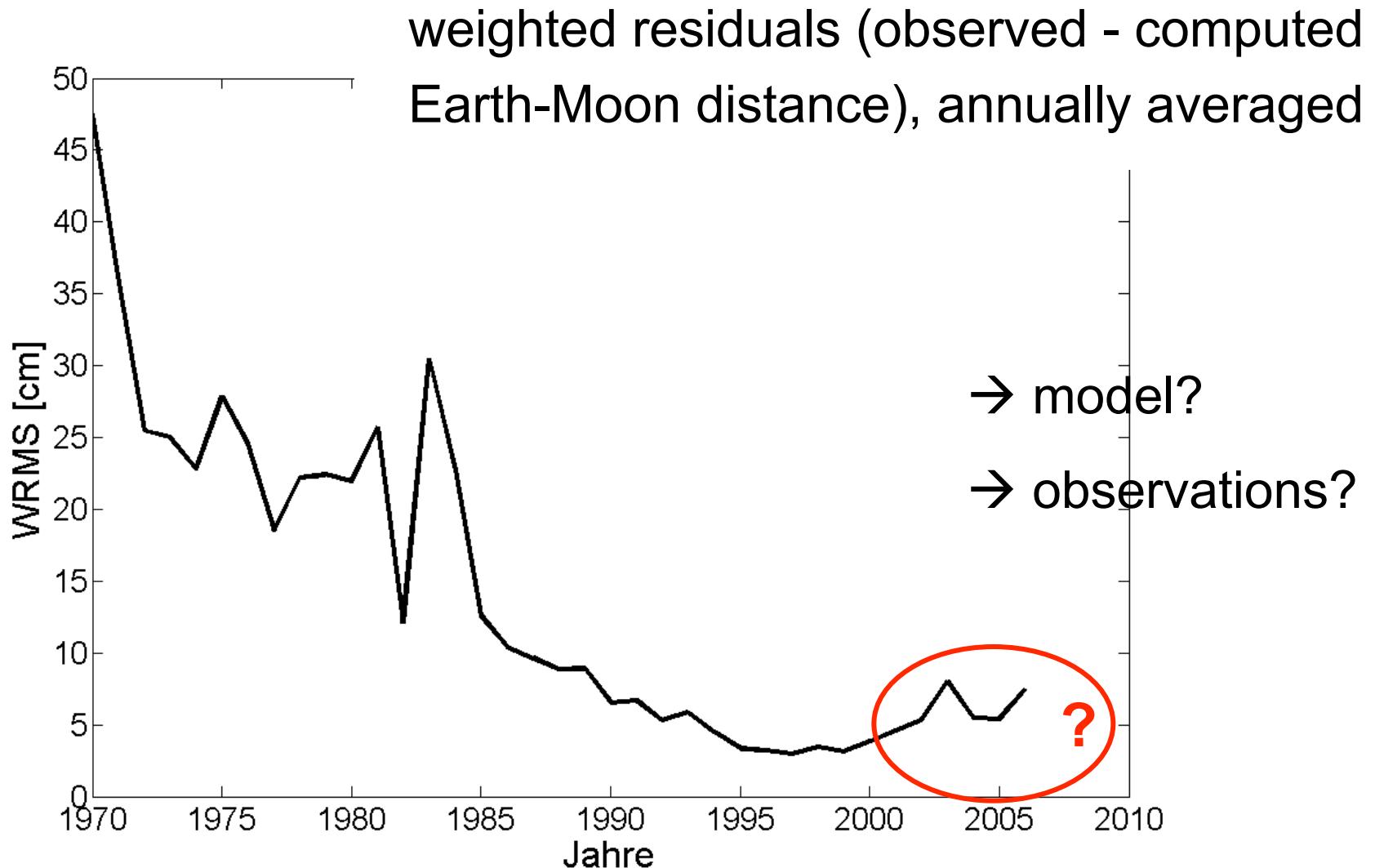


# Distribution of Observations per Synodic Month



- large data gaps near Full and New Moon

# Weighted Annual Residuals



# LLR Results (Theory)

---

## Analysis

- model based upon Einstein's Theory
- least-squares adjustment
- determination of the parameters of the Earth-Moon system (about 180 unknowns, without EOPs)

## Results of major interest

- station coordinates and velocities (ITRF2000) - GGOS
- Earth rotation,  $\sigma = 0.5$  mas (IERS)
- relativity parameters  
(grav. constant, equivalence principle, metric ...)
- ... lunar interior ...

# Example: Gravitational Constant G

---

Investigation of secular and quadratic variations

$$G = G_0 \left( 1 + \frac{\dot{G}}{G} \Delta t + \frac{1}{2} \frac{\ddot{G}}{G} \Delta t^2 \right)$$

Results

$$\frac{\dot{G}}{G} = (2 \pm 7) \cdot 10^{-13} \text{ yr}^{-1}$$

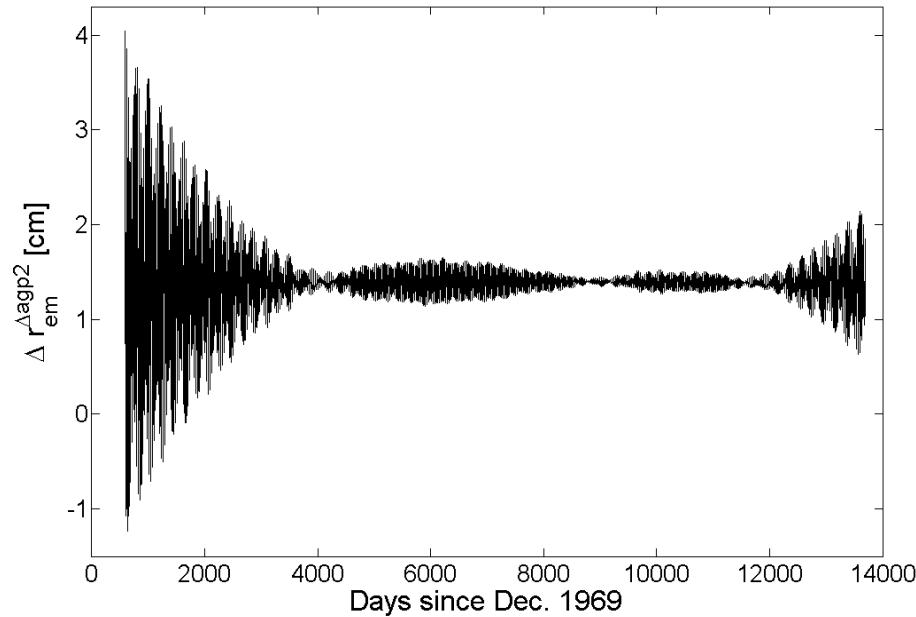
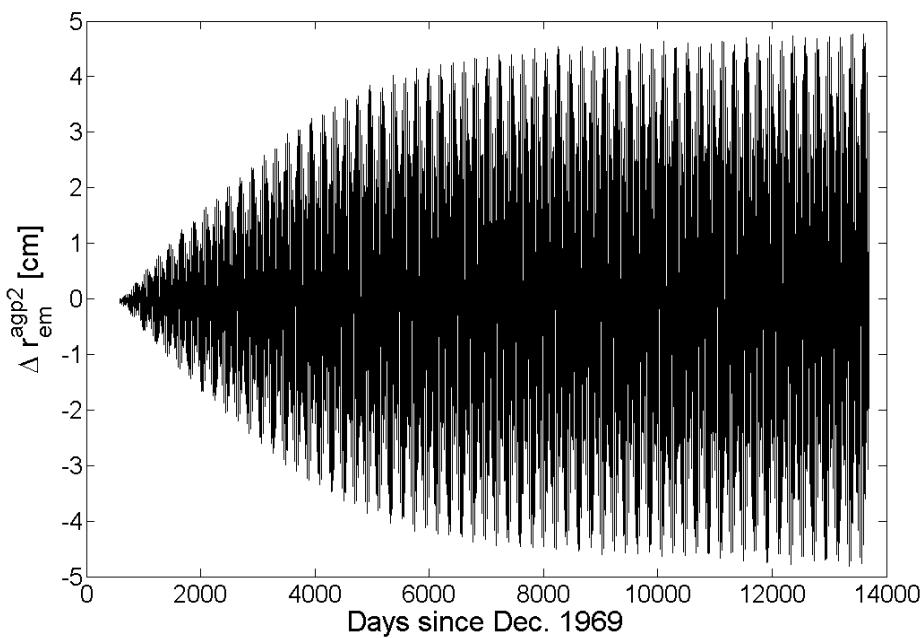
$$\frac{\ddot{G}}{G} = (4 \pm 5) \cdot 10^{-15} \text{ yr}^{-2}$$

# Sensitivity Study for $\ddot{G}$

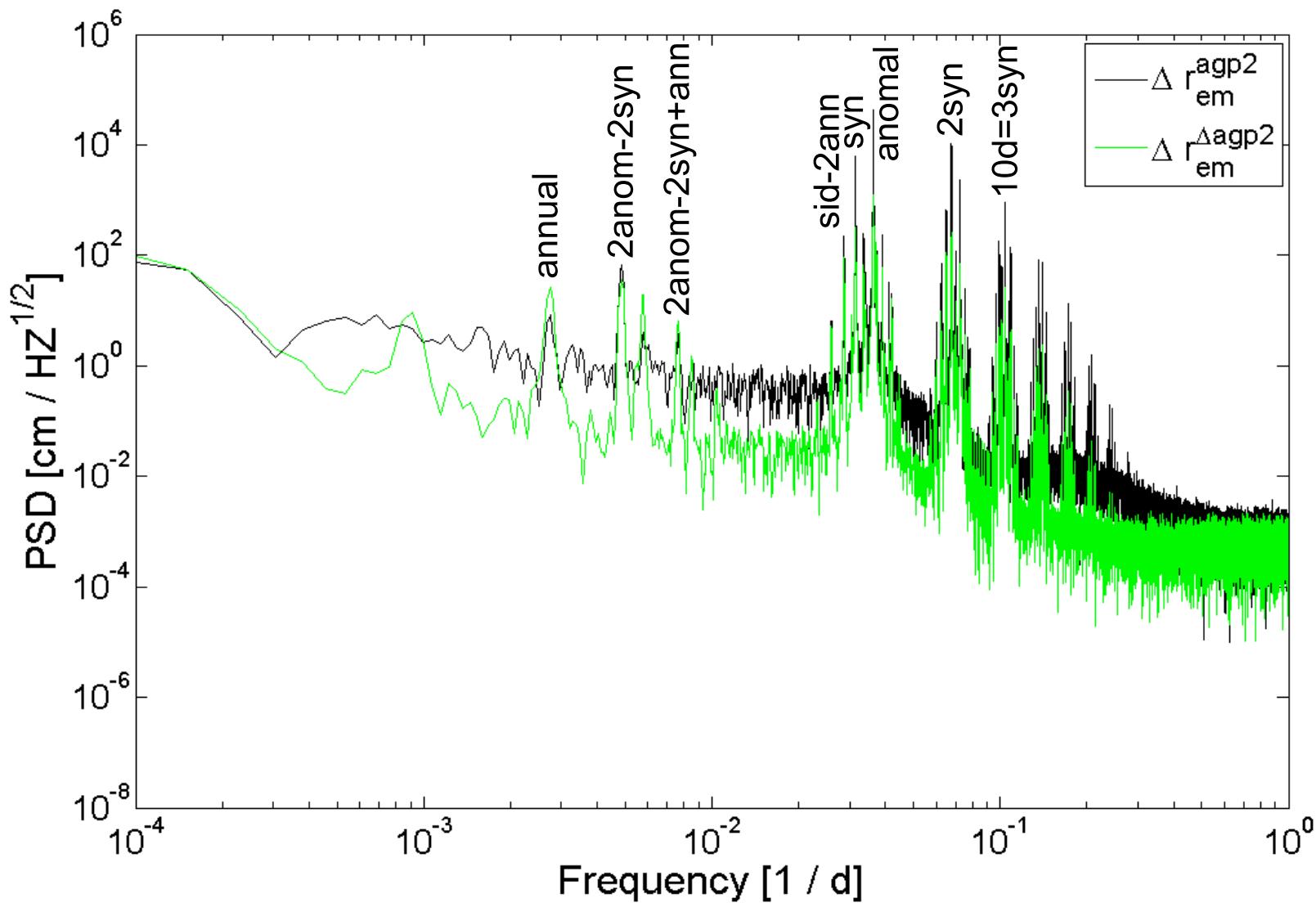
Sensitivity analysis via

$$\Delta r_{em}(\ddot{G}) = \frac{\delta r_{em}}{\delta \ddot{G}} \Delta \ddot{G}$$

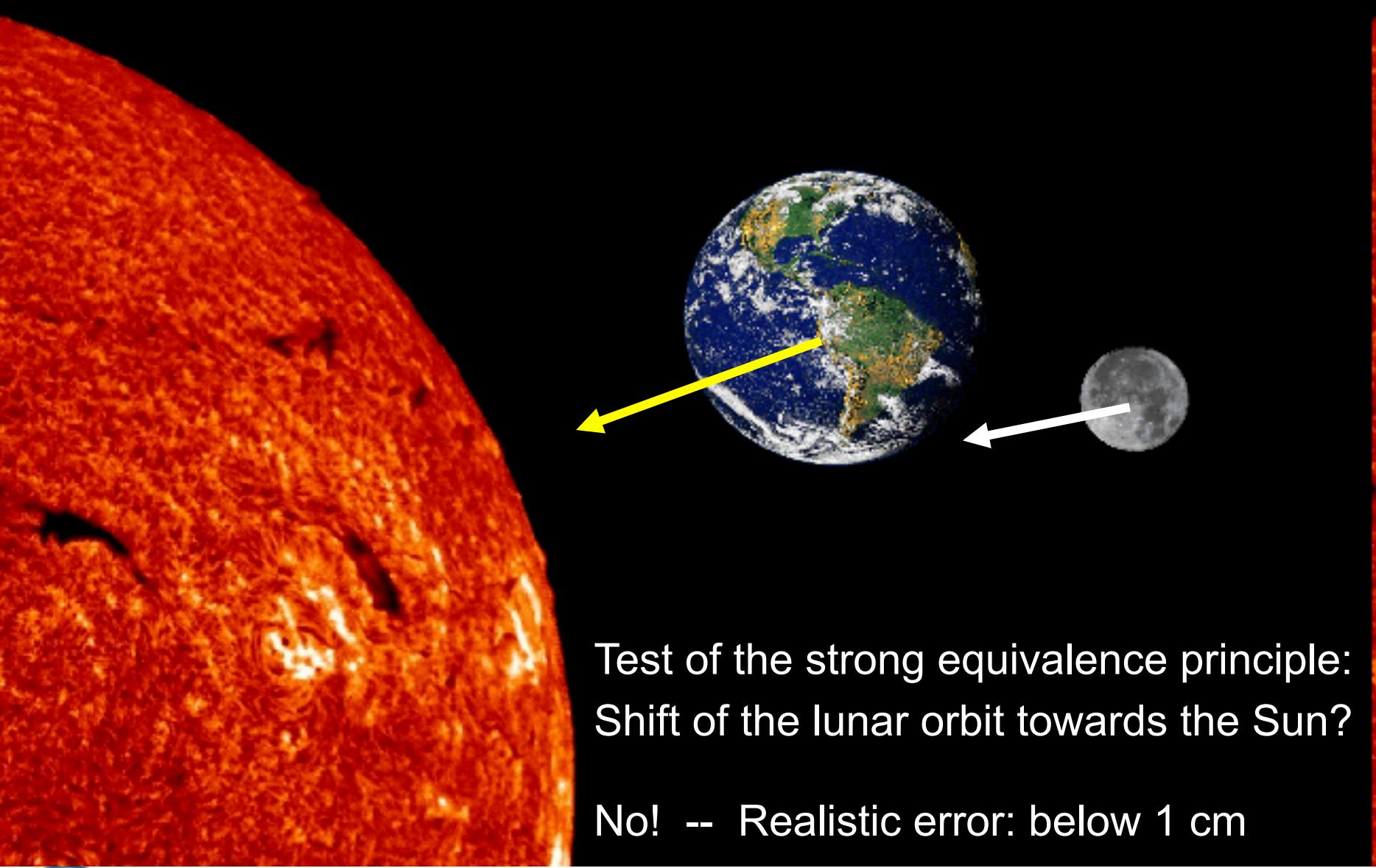
Separation of free and forced terms  $\rightarrow$  two orbit solutions:  
1) perturbed,  
2) un-perturbed  
 $\rightarrow$  difference



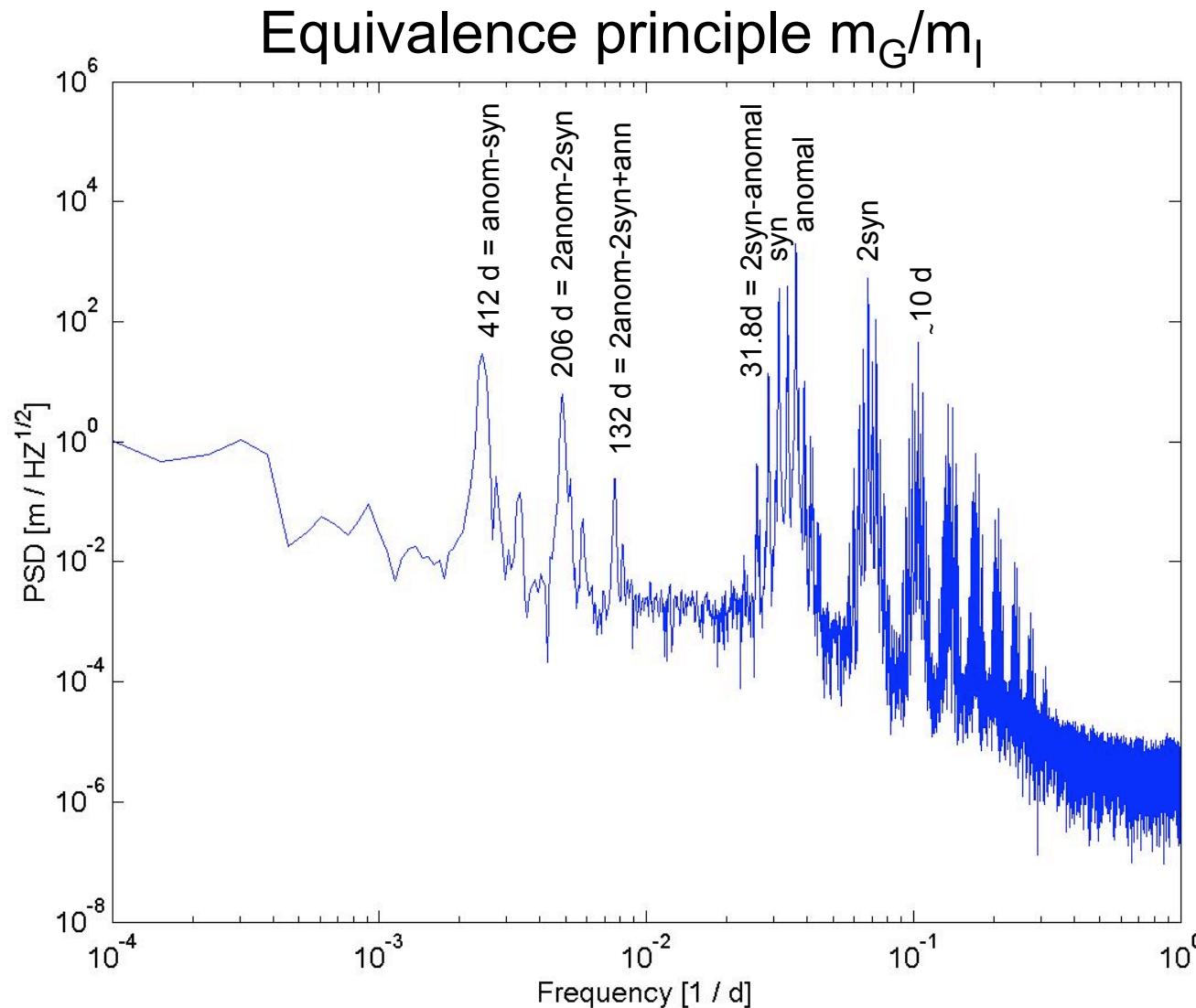
# Corresponding Spectrum



# Example: Nordtvedt Effect

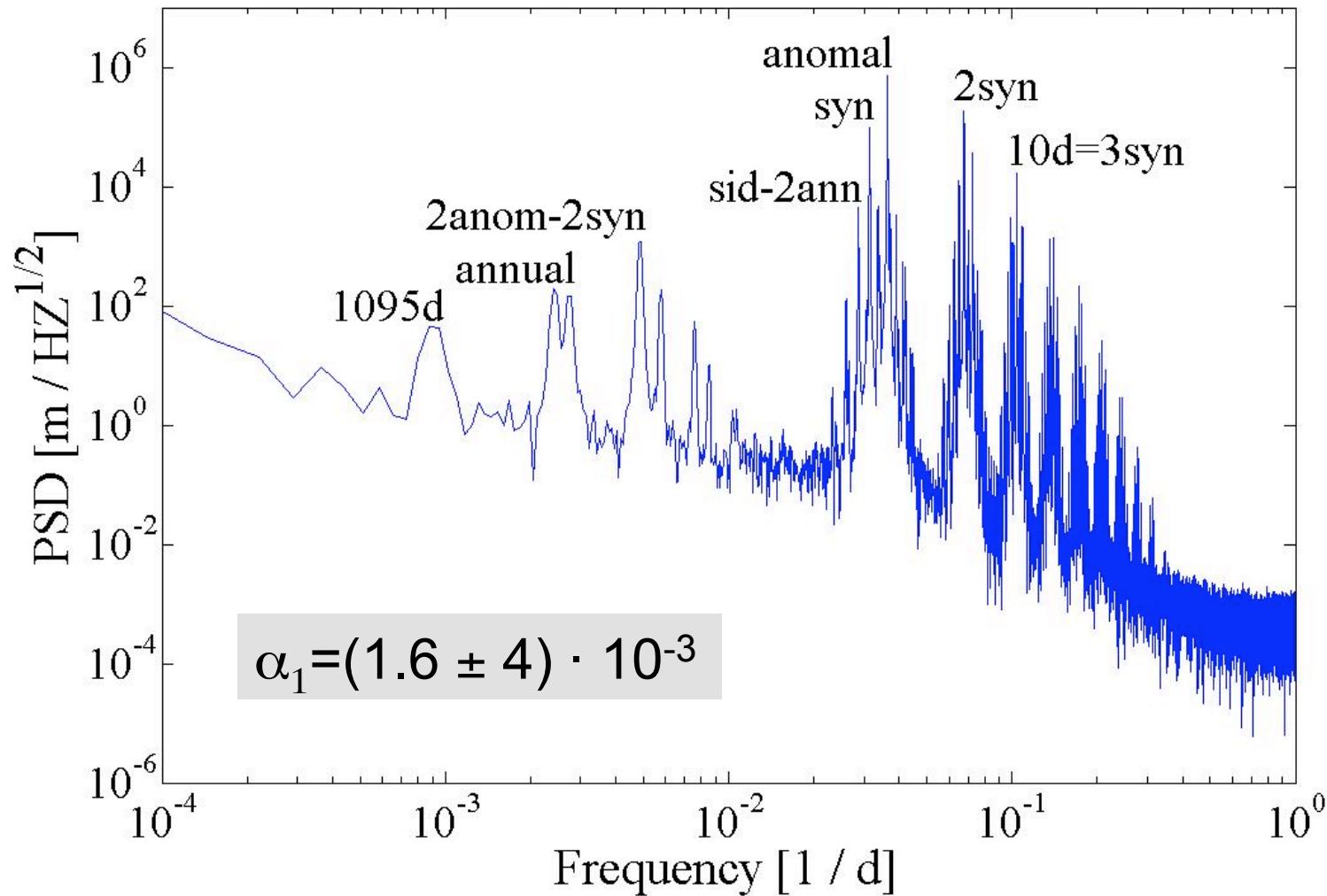


# Relativistic Parameters – Power Spectra (1)



# Relativistic Parameters – Power Spectra (2)

Gravito-magnetic effect (PPN parameter  $\alpha_1$ ) in the solar system



# Results - Relativity

Parameter	Results
Nordtvedt parameter $\eta$ (violation of the strong equivalence principle)	$(6 \pm 7) \cdot 10^{-4}$
time variable gravitational constant $\dot{G}/G [yr^{-1}]$ $\ddot{G}/G [yr^{-2}]$ (→ unification of the fundamental interactions)	$(2 \pm 7) \cdot 10^{-13}$ $(4 \pm 5) \cdot 10^{-15}$
difference of geodetic precession $\Omega_{GP} - \Omega_{deSIT} ["/cy]$ (1.92 "/cy predicted by Einstein's theory of gravitation)	$(6 \pm 10) \cdot 10^{-3}$
metric parameter $\gamma - 1$ (space curvature; $\gamma = 1$ in Einstein)	$(4 \pm 5) \cdot 10^{-3}$
metric parameter $\beta - 1$ (non-linearity; $\beta = 1$ ) or using $\eta = 4\beta - \gamma_{Cassini} - 3$ with $\gamma_{Cassini} - 1 (\sim 10^{-5})$	$(-2 \pm 4) \cdot 10^{-3}$ $(1.5 \pm 1.8) \cdot 10^{-4}$

# Results – Relativity (2)

Parameter	Results
Yukawa coupling constant $\alpha_{\lambda=400\,000\,\text{km}}$ (test of Newton's inverse square law for the Earth-Moon distance)	$(3 \pm 2) \cdot 10^{-11}$
special relativity $\zeta_1 - \zeta_0 - 1$ (search for a preferred frame within special relativity)	$(-5 \pm 12) \cdot 10^{-5}$
influence of dark matter $\delta_{\text{gc}}$ [cm/s <sup>2</sup> ] (in the center of the galaxy; test of strong equivalence principle)	$(4 \pm 4) \cdot 10^{-14}$
preferred frame effects $\alpha_1$ $\alpha_2$ (coupled with velocity of the solar system)	$(-4 \pm 9) \cdot 10^{-5}$ $(2 \pm 2) \cdot 10^{-5}$
preferred frame effect $\alpha_1$ (coupled with dynamics within the solar system)	$(1.6 \pm 3) \cdot 10^{-3}$

# Further Applications

---

## Reference frames

- dynamic realisation of the ICRS by the lunar orbit,  
 $\sigma < 0.01''$  (stable, highly accurate orbit, no non-conservative forces from atmosphere)

## Earth orientation

- Earth rotation (e.g. UT0, VOL)
- long-term nutation coefficients, precession

see Biskupek/Müller,  
session 6, Tuesday

## Relativity

- test of further theories, Lense-Thirring effect

## Combination with other techniques

- combined EOP series and reference frames (GGOS)
- 'Moon' as long-term stable clock

# New Combined Products

---

## Earth orientation

- ◆ UT0 (VLBI)
- ◆ Long-periodic nutation, precession (VLBI, GPS, SLR)

## Celestial reference frame

- ◆ Dynamic realisation of ICRS, ephemeris
- ◆ tie between the lunar network and the radio reference frame (VLBI)

## Gravitational physics parameters

- ◆ Space curvature (VLBI)
- ◆ Lense-Thirring precession (SLR)
- ◆ Others (Grav. constant, equivalence principle metric)

## Lunar interior

# Conclusions

---

**LLR contributes to better understanding of**

- Reference frames (ITRF, dynamic ICRF)
- Earth orientation (IERS)
- Earth-Moon system
- **Relativity**
- Lunar interior
- ...

**... and supports Global Geodetic Observing System**

**In future: new lunar ranging experiment**  
(and combination with other techniques)

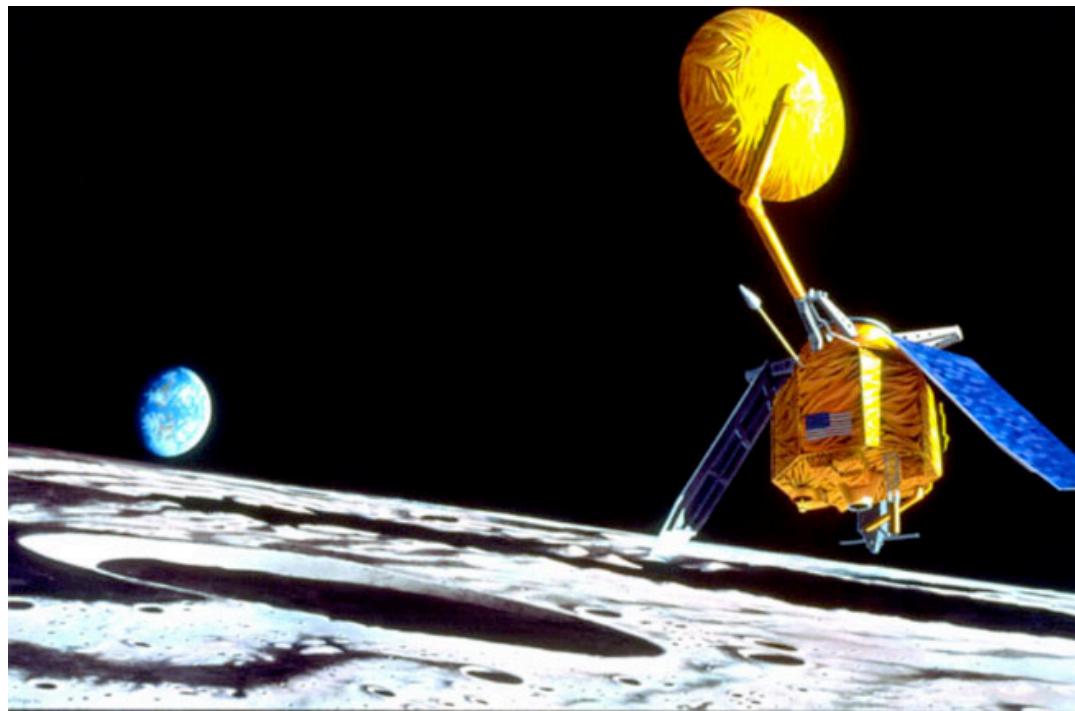
# Future Lunar Missions

---

Deployment of transponders (6 yr lifetime) and new retro-reflectors on the Moon or in lunar orbit

- more observatories
- tie to VLBI (inertial reference frames)

Lunar Reconnaissance Orbiter (LRO)



New high resolution photographs of reflector arrays

- better lunar geodetic network
- lunar maps

# New Ranging Measurements – Why?

---

New data needed to constrain lunar interior structure

- ◆ improve measurements of forced librations
- ◆ measure tidal distortion (amplitude and phase)
- ◆ lunar oscillations as response to large quakes or impacts?

Improve on limits of relativistic effects

- ◆ time variability of the gravitational constant
- ◆ test of strong equivalence principle (Nordtvedt effect)

**Improve the tie between the lunar network and the radio reference frame (VLBI)**

Above goals require more data, more accurate data, and unbiased measurements!